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Adaptive Robust Fuzzy Control for Path Tracking of a Wheeled Mobile Robot

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Abstract: This paper proposes an adaptive robust fuzzy controller for path tracking of a wheeled mobile robot. The robot dynamics including the actuator dynamics is considered in this work. The presented controller is composed of a fuzzy basis function network (FBFN) to approximate an unknown nonlinear function of the robot dynamics, an adaptive robust input to overcome the uncertainties, and a stabilizing control input. The proposed control scheme does not need the robot and the actuator parameters accurately. The stability and the convergence of the tracking errors are proved using the Lyapunov stability theory and the presented adaptive laws. The validity and effectiveness of the proposed control scheme are demonstrated through computer simulations.

Keywords: Fuzzy basis function network, adaptive robust control, robot dynamics, actuator dynamics, uncertainty.

I. INTRODUCTION

In the past, many researches on the path tracking problem for a wheeled mobile robot have been proposed. "Perfect velocity tracking" is put forward in [1] to solve this problem for the kinematic model. In [2], a dynamic controller is presented to integrate with the kinematic controller. However, the controller assumes that the dynamic parameters are completely known. This requirement cannot be carried out in practical situations where it is very difficult to completely obtain the exact parameters of the model. In [3], T. Das and I. N. Kar have proposed an adaptive fuzzy controller to approximate a nonlinear function involving the robot dynamics so that no knowledge of the robot parameters is required. In this paper, although the actuator dynamics has been taken into account, the parameters for actuators has been still required and the same parameters for the right and left actuator models has been also used.

The fuzzy basis function network with a powerful competence for uniformly approximating any nonlinear function over compact input space has been suggested by many researchers [3,4,7]. Although these controllers showed good performance in many simulations and experiments, few literatures on the robustness of the controller to parameter variations and disturbances have been discussed [5,6]. Thus, in this paper, we have established an adaptive robust fuzzy controller so that the mobile robot can track the desired reference path asymptotically with uncertainties. The actuator dynamics for the two wheels of a mobile robot are considered in the dynamic model, and the accurate dynamic parameters of the actuators are not required.

II. MOBILE ROBOT SYSTEM

1. Kinematics and dynamics of a mobile robot

The pose of the robot in the global coordinate frame $\{O, X, Y\}$ is completely specified by the generalized coordinates $q = [x_c \quad y_c \quad \theta]^T$, where x_c and y_c are the coordinates of the point C of center of mass (COM) in the global coordinate frame and θ is the orientation of the local frame $\{C, X_c, Y_c\}$ attached on the robot platform measured from X axis.

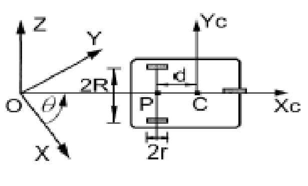

Fig.1. A nonholonomic wheeled mobile robot.

A nonholonomic mobile robot system having a n dimensional configuration space **C** with n generalized configuration variables $(q_1, q_2, ..., q_n)$ and subject to m constraints can be described by

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + \tau_d = B(q)\tau - A(q)\lambda \qquad (1)$$

where $M(q) \in \Re^{n\times n}$ is a symmetric positive definite inertia matrix, $V_m(q,\dot{q}) \in \Re^{n\times n}$ is the centripetal and coriolis matrix, $\tau_d \in \Re^{n\times1}$ denotes bounded unknown disturbances including unmodeled dynamics, $B(q) \in \Re^{n\times r}$ is the input transformation matrix, $\tau \in \Re^{r\times1}$ is the input torque vector, $A(q) \in \Re^{n\times m}$ is the matrix associated with the constraints, and $\lambda \in \Re^{m\times1}$ is the vector of constraint forces.

The complete equations of motion of a mobile robot can be rewritten as

$$\dot{q} = S(q)v(t) \quad (2)$$

$$\overline{M}\dot{v} + \overline{V}_m v + \overline{\tau}_d = \overline{B}\tau \quad (3)$$

where $\overline{M} = S^T MS$, $\overline{V}_m = S^T(M\dot{S} + V_m S)$, $\overline{\tau}_d = S^T \tau_d$, $\overline{B} = S^T B$ and the wheel torque vector $\tau = [\tau_r \quad \tau_l]^T$. The velocity vector $\vec{v} = [v_l \quad w_a]$, v_l and w_a are the linear velocity of the point P along the robot axis and angular velocity, respectively. The $S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}$ can be easily obtained from the following nonholonomic constraint equation, which is $-\dot{x}_c \sin\theta + \dot{y}_c \cos\theta - d\dot{\theta} = 0$ under the pure rolling and non-slipping condition of a mobile robot with two wheels.

Property 1: $\overline{M}$ is uniformly positive definite [2,6].

Property 2: $\left(\dot{\overline{M}} - 2\overline{V}_m\right)$ is skew symmetric [2,6].

Property 3: There exist unknown positive constants $\overline{M}_{\max}$, $\overline{V}_{\max}$, $\overline{\tau}_{d\max}$, and $\overline{B}_{\max}$ such that $\left\|\overline{M}(q)\right\| \le \overline{M}_{\max}$, $\left\|\overline{V}_m(q,\dot{q})\right\| \le \overline{V}_{\max}\left\|\dot{q}\right\|$, $\left\|\overline{\tau}_d\right\| \le \overline{\tau}_{d\max}$, and $\left\|\overline{B}\right\| \le \overline{B}_{\max}$ [6].

2. Dynamic model of a mobile robot including actuator dynamics

The actuator dynamics with the same parameter for two DC motors is brought out in [3]. The electrical part of the actuator is taken into account, and it is considered that two DC motors have the different parameters in this paper. The dynamics equation including the actuator dynamics can be written as

$$\overline{M}\dot{v} + \overline{V}_m v + \overline{B}D_v v + \overline{\tau}_d = Hu \quad (4)$$

where $H = \overline{B}D_u$, $\overline{B} = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ R & -R \end{bmatrix}$

$D_u = diag\left(\frac{N_r K_{Tr}}{R_{ar}}, \frac{N_l K_{Tl}}{R_{al}}\right)$, $X = \frac{1}{r}\begin{bmatrix} 1 & R \\ 1 & -R \end{bmatrix}$, and

$D_v = diag\left(\frac{N_r^2 K_{Tr} K_{br}}{R_{ar}}, \frac{N_l^2 K_{Tl} K_{bl}}{R_{al}}\right) \cdot X$.

$diag(a_1, a_2)$ represents a 2×2 diagonal matrix of the diagonal elements a_i. N_r and N_l are the gear ratios of the right and left motors. K_{Tr} and K_{Tl} are the motor torque constant, R_{ar} and R_{al} are the electric resistance, K_{br} and K_{bl} is the counter electromotive force coefficient of left and right motors, respectively. R and r are shown in Fig. 1. $u = [u_r \quad u_l]^T$ is the actuator input voltage vector and used as the control input instead of the wheel torque vector τ. In this robot model (4), the motor inductances are neglected.

III. DESIGN OF AN ADAPTIVE ROBUST FUZZY CONTROLLER

If we consider only the kinematic model (2) for the mobile robot and assume that there is "perfect velocity tracking" so that $v = v_c$, then the kinematic model is asymptotically stable to a reference trajectory.

The tracking error is expressed in the basis of a frame fixed on the mobile robot as

$$E_p = T_e(q_r - q)$$

$$E_p = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (5)$$

The velocity control input that achieves the asymptotic tracking for the kinematic model is

$$\vec{v}_c = \begin{bmatrix} v_{lc} \\ w_{ac} \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 + k_1 e_1 \\ w_r + k_2 v_r e_2 + k_3 v_r \sin e_3 \end{bmatrix} \quad (6)$$

where $k_1 > 0$, $k_2 > 0$, and $k_3 > 0$ are design parameters [1]. This velocity control input (6) can guarantee that E_p converges to zero asymptotically as time goes to infinity, when $d = 0$.

Once the auxiliary velocity control input $v_c(t) \in \Re^{n-m}$ are obtained for the asymptotically stable kinematic steering system, we need to design an actuator input voltage $u(t)$ to guarantee the robust tracking despite unknown robot and actuator dynamic parameters and external disturbances.

The auxiliary velocity tracking error is denoted as

$$e_c = v_c - v \quad (7)$$

Differentiating (7) with respect to time and substituting it into (4), the robot dynamics can be expressed as

$$\overline{M}\dot{e}_c = -\overline{V}_m e_c + \overline{M}\dot{v}_c + \overline{V}_m v_c + \overline{B}D_v v - Hu + \overline{\tau}_d \quad (8)$$

where the nonlinear function containing the dynamic parameters of the robot and actuator parameters is

$$f(x) = \overline{M}\dot{v}_c + \overline{V}_m v_c + \overline{B}D_v v \quad (9)$$

which can be approximated using a following FBFN [4]

$$f(x) = \begin{bmatrix} \sum_{j=1}^{N_1} p_{vj}(x)\theta_{vj} \\ \sum_{j=1}^{N_2} p_{wj}(x)\theta_{wj} \end{bmatrix} + \begin{bmatrix} \varepsilon_v(x) \\ \varepsilon_w(x) \end{bmatrix} = R(v_c, \dot{v}_c, v)\Phi + \varepsilon(x) \quad (10)$$

$$p_{vj_1}(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^{j_1}}(x_i)}{\sum_{j_1=1}^{N}\prod_{i=1}^{n} \mu_{A_i^{j_1}}(x_i)},\ p_{wj_2}(x) = \frac{\prod_{i=1}^{n} \mu_{\overline{A}_i^{j_2}}(x_i)}{\sum_{j_2=1}^{N}\prod_{i=1}^{n} \mu_{\overline{A}_i^{j_2}}(x_i)},$$

$$j_1 = 1,, 2, \ldots, N_1,\ j_2 = 1,, 2, \ldots, N_2 \quad (11)$$

where $x=(v_c,\dot{v}_c,v)$ is the input variable of fuzzy basis function $p_{vj}(x)$ and $p_{wj}(x)$ are called the fuzzy basis functions which corresponds to fuzzy IF-THEN rules, θ_{vj} and θ_{wj} are free parameters, $R(v_c,\dot{v}_c,v)\in\Re^{2\times[2*\max(N_1,N_2)]}$ is called a fuzzy basis function matrix, $\Phi=\left[\theta_v^T \quad \theta_w^T\right]^T\in\Re^{2*\max(N_1,N_2)}$ is the desired parameter vector, which is an unknown constant vector to be determined to closely approximate the nonlinear function. $\varepsilon(x)$ is the approximation error vector. $\mu_{A_i^j}(x)$ and $\mu_{A_i^j}(x)$ are the Gaussian membership functions, defined by $\mu_{A_i^j}(x_i)=a_i^j\exp\left[-\frac{1}{2}\left(\frac{x_i-\bar{x}_i^j}{\sigma_i^j}\right)^2\right]$, where $a_i^j,\bar{x}_i^j$ and σ_i^j are real valued parameters with $0<a_i^j\le 1$.

Property 4: There exist unknown positive constants $\bar{\theta}_1,\bar{\theta}_2,\bar{\theta}_3$ and $\bar{\theta}_4$ such that

$$\|\varepsilon(x)\|\le\bar{\theta}_1\|\dot{v}_c\|+\bar{\theta}_2\|\dot{q}\|\|v_c\|+\bar{\theta}_3\|w\|\|v_c\|+\bar{\theta}_4\|v\|=\rho_e\,. \quad (12)$$

The above property can be easily obtained from Property 3.

It is considered that the input matrix H including the actuator parameters is unknown. Hence the control input is chosen as follow,

$$u=\hat{H}^{-1}\bar{u} \quad (13)$$

where $\hat{H}$ is the guessed nominal parameter matrix of H and $\bar{u}$ is an adaptive fuzzy control input.

Substituting (13) into (8), the closed-loop error dynamics for e_c is

$$\bar{M}\dot{e}_c=-\bar{V}_m e_c+f(x)+(I-H\hat{H}^{-1})\bar{u}-\bar{u}+\bar{\tau}_d \quad (14)$$

Assumption 1: There exist a positive constant C_0 such that

$$\left\|I-H\hat{H}^{-1}\right\|\le C_0<1 \quad (15)$$

Theorem 1: Under Assumption 1, if the following control and adaptation laws (16)~(19) are applied to the mobile robot system (2) and (4), then e_c converges to zero asymptotically when $v_r>0$. Therefore, in the case that $d=0$, by the velocity control (6), e_1, e_2, and e_3 converge to zero asymptotically. Finally, $x\to x_r$, $y\to y_r$ and $\theta\to\theta_r$ as $t\to\infty$.

$$u=\hat{H}^{-1}\bar{u}=\hat{H}^{-1}\left(\hat{f}+u_s\right),\quad \hat{f}(x)=R(v_c,\dot{v}_c,v)\hat{\Phi}\,, \quad (16)$$

$$u_s=\hat{\rho}\frac{e_c}{\|e_c\|}+Ke_c\,,\quad \hat{\rho}=\hat{\Theta}^T\psi \quad (17)$$

$$\dot{\hat{\Phi}}=\Gamma_\phi R^T e_c\,,\qquad \dot{\hat{\Theta}}=\Gamma_\theta\psi\|e_c\|\in\Re^7 \quad (18)$$

$$\psi=\left[1,\|\dot{v}_c\|,\|\dot{q}\|\|v_c\|,\|w\|\|\dot{v}_c\|,\|v\|,\|R\hat{\Phi}\|,\|e_c\|\right]^T \quad (19)$$

where $\hat{f}(x)$ is an estimate of the nonlinear robot function $f(x)$ estimated by using a FBFN, and u_s is the adaptive robust and stabilizing control term, $\hat{\Phi}$ is an estimate updated by (18). The dimension of the estimate $\hat{\Phi}$ is determined by the fuzzy basis functions and fuzzy rules used in the FBFN. $\hat{\Theta}$ is an estimate of real norm-bound values Θ and updated by (18). ψ is the bounding function obtained in the stability proof. K, Γ_ϕ and Γ_θ are positive constant diagonal gain matrices. ■

Proof: Consider the Lyapunov function candidate:

$$V=V_1+V_2+V_3 \quad (20)$$

where $V_1=\frac{1}{2}(e_1^2+e_2^2)+\frac{1}{k_2}(1-\cos e_3)$,

$V_2=\frac{1}{2}e_c^T\bar{M}e_c+\frac{1}{2}\tilde{\Phi}^T\Gamma_\phi^{-1}\tilde{\Phi}$, and $V_3=\frac{(1-C_0)}{2}\tilde{\Theta}^T\Gamma_\theta^{-1}\tilde{\Theta}$.

The time derivative of V_1 in (20) is as follows by (6).

$$\dot{V}_1=e_1\dot{e}_1+e_2\dot{e}_2+\frac{1}{k_2}\dot{e}_3\sin e_3=-k_1e_1^2-\frac{k_3}{k_2}v_r\sin^2 e_3\,. \quad (21)$$

After differentiating V_2 with respect to time, substituting it into the error dynamics (14), adopting Property 2, and applying it to (16)~(18), we obtain

$$\dot{V}_2=e_c^T\varepsilon-e_c^Tu_s+e_c^T(I-H\hat{H}^{-1})(R\hat{\Phi}+u_s)+e_c^T\bar{\tau}_d \quad (22)$$

where $\tilde{\Phi}=\hat{\Phi}-\Phi$.

From Property 3, Property 4 and Assumption 1, the boundedness of (22) can be obtained as

$$\dot{V}_2\le\bar{\rho}_e\|e_c\|+C_o\|e_c\|\|R\hat{\Phi}\|+C_o\|e_c\|\|u_s\|-e_c^Tu_s \quad (23)$$

where $\bar{\rho}_e=\rho_e+\bar{\tau}_{d\max}$.

From the definition of u_s in (17), it can be inferred that

$$\|u_s\|\le\hat{\rho}+K_m\|e_c\| \quad (24)$$

where $\|K\|\le K_m$ and K_m is a positive constant.

Thus we can obtain the following inequality.

$$\dot{V}_2\le\bar{\rho}_e\|e_c\|+C_o\|e_c\|\|R\hat{\Phi}\|+C_oK_m\|e_c\|^2+(C_o-1)\hat{\rho}\|e_c\|-e_c^TKe_c \quad (25)$$

Differentiating V_3 with respect time, we obtain

$$\dot{V}_2+\dot{V}_3\le-e_c^TKe_c-\hat{\rho}(1-C_o)\|e_c\|+\rho(1-C_0)\|e_c\|+(1-C_o)\tilde{\Theta}^T\Gamma_\theta^{-1}\dot{\hat{\Theta}} \quad (26)$$

where $\rho=\frac{1}{(1-C_0)}\left[\bar{\rho}_e+C_o\|R\hat{\Phi}\|+C_oK_m\|e_c\|\right]$.

Since $\tilde{\rho}=\hat{\rho}-\rho=\hat{\Theta}^T\psi-\Theta^T\psi=\tilde{\Theta}^T\psi$, then

$$\dot{V}_2+\dot{V}_3\le-e_c^TKe_c-\tilde{\Theta}^T\psi(1-C_o)\|e_c\|+(1-C_0)\tilde{\Theta}^T\Gamma_\theta^{-1}\dot{\hat{\Theta}}\,. \quad (27)$$

By choosing the adaptation law (18) and using (21) and (27), we can conclude that

$$\dot{V}=\dot{V}_1+\dot{V}_2+\dot{V}_3\le-k_1e_1^2-\frac{k_3v_r\sin^2 e_3}{k_2}-e_c^TKe_c<0\,. \quad (28)$$

It can be easily found that the velocity tracking errors e_c converge to zero asymptotically when $v_r>0$. In the case that $d=0$, by the velocity control (6), e_1, e_2, and e_3 converge to zero asymptotically. □

IV. SIMULATION RESULTS

Simulation results for path tracking control of a mobile robot with two wheels against the uncertainties are presented. The desired reference trajectory is $x_r(t) = \cos(\frac{2\pi t}{50}),\ y_r(t) = \sin(\frac{2\pi t}{50}),\ \theta_r(t) = \tan^{-1}\left(\frac{\dot{y}_r}{\dot{x}_r}\right)$. The initial posture of the reference trajectory is $q_r(0) = [1 \quad 0 \quad \pi/2]^T$ and the initial posture of the robot is chosen as $q(0) = [1.2 \quad 0.5 \quad \pi/3]^T$

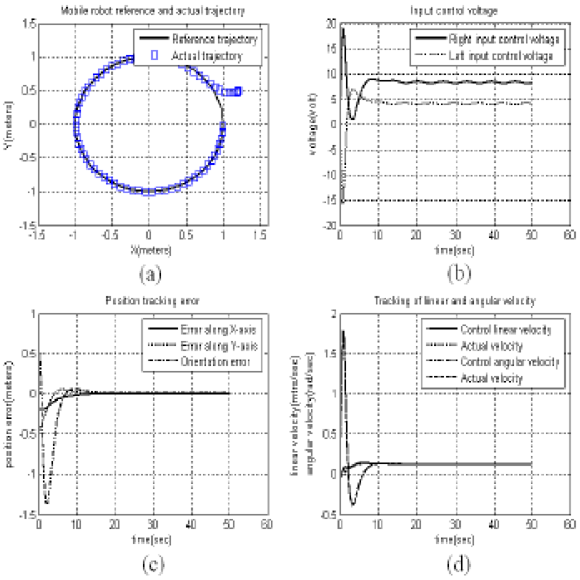

Fig.2. (a) Path tracking of the robot; (b) Control input voltages; (c) Position tracking errors; (d) Tracking of linear and angular velocities.

In the simulation, the system dynamics parameters are set as $N_r = 100$, $N_l = 80$, $K_{br} = 0.0274$, $K_{bl} = 0.025$, $K_{tr} = 0.0274(Nm/A)$, $K_{tl} = 0.025(Nm/A)$, $R_{ar} = 4\Omega$, $R_{ar} = 3\Omega$, $m = 10kg$, $I = 5kg \cdot m^2$, $R = 0.2(m)$, $r = 0.05(m)$, $d = 0.01(m)$.

The knowledge of the system dynamic parameters are set to 70% of the real values. The disturbance is unknown in the control system and generated in this simulation by $\tau_d = \sin(t)$. The simulation is also performed with considering parameter variations. During the total control process, mass and inertia of the robot vary three times as follows: 1) $0 \le t < 20$, $m = 10, I = 5$; 2) $20 \le t < 30$, $m = 20, I = 10$; 3) $30 \le t < 40$, $m = 25, I = 15$, 4) $40 \le t \le 50$, $m = 30, I = 20$.

In the presented FBFN, the number of fuzzy rules here is simply chosen as ten rules. For each input variable, three Gaussian basis functions are used.

The gains used in the controller are chosen as $k_1 = 0.25$, $k_2 = 50$, $k_3 = 10$, $K = 50I_2$, $\Gamma_\phi = 5000I_{10}$, $\Gamma_\theta = 0.00005I_7$ where $I_n \in \Re^{n \times n}$ is an identity matrix.

From these simulation results shown in Figs. 2(a)~(d), it is found that the proposed control scheme is valid and robust against parameter variations and disturbances.

V. CONCLUSIONS

In this paper we have developed an adaptive robust fuzzy controller for a wheeled mobile robot that asymptotically tracks a desired reference path in the presence of uncertainties. The stability and the convergence of the tracking errors have been guaranteed based on the Lyapunov function theorem. The different parameters for two actuator models have been considered in the proposed controller. The proposed control scheme does not require the accurate dynamic parameters of the robot and the actuator parameters. The effectiveness and robustness of the proposed controller has been shown through the simulation.

In addition to this work, a control scheme considering the inductances of the actuators is left to a further study.

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